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COMPRESSION OF DATA AND ACCURACY OF RESTORATION OF ANALOG SIGNALS

V. P. Yevdokimov

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16. Abstract

The article discusses the restoration of the shape of analog signals according to discrete readings, passed through polynomial compression devices. Various models of the signal are employed, differing in the shape of the correlation function. The accuracy criteria are the mean square error of restorations in the middle of the interval between responses, at the edge of the interval and the average over the interval.

Calculations are performed of the errors in the restoration of the shape and the compression coefficients for different numbers of interrogation points in the correlation interval and the transitional threshold in the compression structures.

The analysis was carried out with the aid of simulation of the signals and compression devices on a computer.

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COMPRESSION OF DATA AND ACCURACY OF RESTORATION OF ANALOG SIGNALS

V. P. Yevdokimov

I. Statement of the Problem /3*

In telemetry systems with time separation of the channels, the choice of the interrogation frequency for the individual channels is usually governed by a requirement for ensuring accuracy of measurement with the most rapid changes in the sensor signals. The excess which is thus obtained with respect to the interrogation frequency at slower variations in the signal decreases with the aid of data compression devices. The most widespread of these devices are predictors and interpolators. The criterion for the reduction of the excess in practice is usually considered to be the ratio of the number of selections transmitted over the communication line per unit time with cyclic interrogation of the sensor to the average number of selections per unit time at the output of the compression device (compression coefficient) with set requirements as to the accuracy of data restoration. The restoration of the shape of the analog signals following reception is conducted using uniform sequences of discrete responses, obtained by using an algorithm, the reverse algorithm of compression. The selections which turn out to be excessive as a result of the operation of the compression device are replaced in the estimate whose accuracy depends upon the threshold of the compression device. The error in the restoration of the shape of the analog signal between measurements begins to depend not only upon the shape of the signal, the /4

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*Numbers in the right-hand margin indicate pagination in the foreign text.

method of approximation, the interrogation frequency, but also the algorithm of the operation of the compression device, its threshold and compression coefficient.

The goal of the present paper was to determine these relationships.

In [1] it was shown that for a wide class of correlation functions the signal of the section-line approximation of the shape between the regular recordings (combination of adjacent recordings by straight lines) gives results that are practically the same with respect to restoration error as the optimum linear restoration. In this work, although we shall be using a data compression device which introduces an error into part of the calculations of the restored regular sequence, use is also made of the method of section-line approximation of the shape of the signal between recordings.

The criteria for accuracy will be considered to be the mean square error of restoration at the center, at the edge of the time interval between recordings and the average over the interval.

II. Models of the signal, method of calculating errors and compression coefficients, compression algorithms.

As models of the signal, we shall examine the normal stationary random processes with zero average, having the following correlation functions:

1. $R(\tau) = \delta^2 \exp(-d|t|)$, $\alpha = 4F_{eff}$

corresponds to quiet noise, transmitted through a RC filter with an effective bandpass F_{eff}

2) $R(\tau) = \sigma^2 \exp(-\alpha\tau^2)$, $\alpha = 4\pi F_{eff}^2$

corresponds to the white noise passed by a filter with a Gaussian frequency characterstic and an effective band--*. These two forms were selected as the two extreme representatives of the class of correlation functions for which analytical calculation of errors is carried out in [1] (without the compression device). /5

In addition, the generalized frequency of discretization employed is equal to the ratio of the interrogation frequency F_0 to the doubled effective width of the signal spectrum and showing the number of interrogation points in the interval of correlation of the process:

$$r = \frac{F_0}{2F_{eff}} = \frac{T_{thresh}}{T_0}$$

Calculating the errors in restoration and the coefficients of compression were carried out with the aid of a simulation of random processes and the above mentioned correlation function on a digital computer. For the process with correlation functions of the first type, with the aid of a recurrent algorithm, described in [2], the long realization of the discrete random process was formulated, the correlation between the samples corresponding to the continuous random process with 512 interrogation points in the correlation interval. For the correlation function of the second type, the formed discrete sequence [2] corresponded to the random process with 128 interrogation points in the correlation interval. Obtaining random processes with reduced frequencies of time discretization was accomplished

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*[Translator's note: Blank in original.]

by thinning the original random sequences.

The correspondence of the discrete random sequences obtained to the given random processes was checked on the computer during the basic calculation by computation of the reproduction errors by means of a section-line approximation with various interrogation frequencies without compression taken into account. Inasmuch /6 as the restoration error is unambiguously dependent upon the form of the correlation function, the practical coincidence obtained for the results of simulation with the calculation carried out in [1] for these same errors on the basis of an analytical formula made it possible to conclude that there was a good quality model of the process employed.

As the polynomial compression devices, we use a digital computer to construct models of a predictor of zero order with a floating aperture and a fan-shaped interpolator of the first order (ZOP and IFO), operating using the algorithms given below.

Zero Order Predictor

1. Reference sample X_i is transmitted.

2. Averaging of the moduli of the sample difference X_{i+k} is carried out (where $k = 1$ initially) and the reference sample with threshold Z.

3. If $|X_{i+k}-X_i|$ is less than Z, sample X_{i+k} will be considered excessive, K will increase to 1 and step 2 will be repeated. If $|X_{i+k}-X_i| \geq Z$, the sample with the number i+K will be considered reference and the step 1 will be repeated replacing i by i+k.

The horizontal straight lines which run to the right of each important sample determine the estimates of all intermediate samples and the points between them.

First Order Interpolator

1. A sample X_i is transmitted.

2. Using sample X_{i+1}, an upper $\hat{X}_b$ and a lower $\hat{X}^h$ is constructed

for the estimate of the sample X_{i+2} is constructed by drawing straight lines through X_i, $X_{i+1}+Z$, $X_{i+1}-Z$.

3. If the sample X_{i+k} (where k = 2 initially) lies beneath /7 the estimates, we consider it to be excess.

4. If $\hat{X}^b_{i+K}$ or $\hat{X}^H_{i+K}$ is located between points $X_{i+K}+Z$ and $X_{i+K}-Z$, we can draw a line of the estimate for obtaining the upper or the lower estimate of the sample X_{i+K+1}. If $\hat{X}^B_{i+K} > X_{i+K}+Z$, we can measure the slope of the upper estimate for X_{i+K+1}, extending it through X_i and $X_{i+K}+2$.

5. We then increase k to unity and repeat step 3.

6. If sample X_{i+k} is greater than or equal to the upper estimate or less than or equal to the lower estimate we consider it to be real and return to step 1, replacing i in i+K-1.

Straight lines connecting the transfered samples determine the estimates of all intermediate samples and the points betwen them.

3. Results of Calculations and Conclusions

The results of the simulation of the signals [illegible] are discussed below in the form of tables and graphs.

In calculating the mean square errors of the restored forms of the signal, the normalization of the calculated values are derived according to the side of variation of the signal 6δ (and 3δ).

$$E(\varepsilon)=\frac{\sqrt{\overline{E^2(\varepsilon)}}}{6}\ ,\quad E_{av}=\frac{\sqrt{\overline{E^2_{av}}}}{\varepsilon}$$

The thresholds of the compression devices were assumed to be equal to 1, 3, 5, and 10% of the scale 6δ (i.e., 0.06 / 0.18, 0.3, 0.6 δ).

Averaging over the interval of the error we can calculate the approximate integrals using the formula of parabolas /8 over eight points inside each interval between samples. In this connection, we did not calculate the average in the interval of the error with r = 256 and 128 for a process with a correlation function of the first type and with r = 64 and 32 for a process with correlation function of the second type.

In Tables 1, 2, 3, and 4 we have presented the results of the calculation of the mean square errors of restoration in the middle of the interval--$E_{0.5}$, the average over the interval--E_{av} and at the edge of the interval--E_{edge} as a function of the frequency of interrogation and the threshold of the compression device. For convenience in comparison, these same tables show the values of the errors in the absence of a compression device (the error at the edge of the interval is equal to zero).

By means of these tables, we have plotted the relationships between the errors at the center and at the edge of the interval between samples on the frequency of interrogation at the threshold as a parameter shown in Figures 1, 2, 3, and 4.

The representation of errors $E_{0.5}$ and E_{edge} in these figures shows a general law: as the frequency of interrogation increases the mean square error at the edge and distance at the center of the interval tend toward the same limit.

It is interesting to note that for a process with a Gaussian frequency characteristic (second type), with functioning of the predictor, the errors tend with an increase in r to a value $2Z/\sqrt{12}$, shown in Figure 2 and Figure 3 by a dashed line, corresponding to the mean square deviation of the distribution of the error probability in the interval $\pm Z$. However for the interpolator the limit is the threshold value of Z. Inasmuch as we know the maximum value of Z for the error at the edge of the interval, we can see that the mean square value of this error tends toward a maximum error as r increases. The distribution of the probability of the error then clearly develops into a discrete distribution with identical values $P = 0.5$ at the edges of the interval $\pm Z$. The errors in the middle of the intervals begin to behave /9
similarly.

For a process of correlation function of the first type, the limit of errors $E_{0.5}$ and E_{edge} are greater than the mean square value of the uniform distribution but less than the maximum value Z.

The errors in restoration $E_{0.5}$ and E_{av} for the correlation function of the second form on any threshold have a minimum corresponding to some interrogation frequency.

It is interesting to note that the same assumption regarding the error in excess samples (threshold Z) in the case of an interpolator, leads to an error which is approximately twice as great in $E_{0.5}$ than for the predictor.

Figures 5 and 6 show the relationships of errors $E_{0.5}$ on the threshold with an interrogation frequency as a parameter. The characteristic feature of these graphs is the practically linear dependence of the errors upon the threshold value, especially clearly evident for a process with a Gaussian form of the spectrum.

Tables 5 and 6 show the results of a calculation of coefficients of compression for various thresholds and interrogation frequencies. In these calculations, the procedure information was not taken into account. At low thresholds, the coefficients of compression are small even with high interrogation frequencies. As the interrogation frequency increases with $V > 3\%$ the coefficient of compression increases sharply. The tables also show the practically linear change in the coefficient of compression with increase in threshold for each interrogation frequency. This relationship may turn out to be useful in calculating the buffer memory in transmission systems that use a compression device.

In Table 7, we have displayed the results of a comparison of coefficients of compression obtained when using a predictor of the
zero order, a device with a variable and adjustable interroga- /10
tion frequency, a zero-order interpolator with given requirements as to mean square error in the middle of the interval, calculated on the basis of the graph provided, and the tables for the second type of correlation function. For a predictor and interpolator, we have examined the cases of the variable thresholds for retention of the constant error of value with a change in interrogation frequency and a fixed threshold at which the error is always less than a given value. The table shows the undesirability of establishing the threshold due to the low gain with respect to compression.

At all interrogation frequencies, the interpolator is approximately 1-1/2 times more efficient than a device with a variable interrogation frequency at 1.5 to 3 times more effective than a predictor (the gain is increased with increasing interrogation frequency). A device with variable frequency gives a gain in comparison with the predictor only with an increase in the interrogation frequency.

All of the results shown indicate that significant compression coefficients can be expected only at high threshold values relative to the scale of change in signal or with high interrogation frequencies. This indicates that the nonstationary nature of the signals, which leads to a relative increase in threshold, can lead to a much greater compression coefficient than nonstationary conditions expressed in a change in the width of the signal spectrum.

TABLE

Predictor	$R(\tau) = exp(\alpha/\|\tau\|)$									
τ / Z%	1	2	4	8	16	32	64	128	256	E%
w/out comn	15,2	11,5	8,33	5,82	4,16	2,91	2,06	1,43	1,05	$E_{0,5}$
1	15,2	11,5	8,33	5,82	4,16	8,92	2,1	1,45	1,09	
3	15,2	11,55	8,33	5,86	4,2	3,0	2,25	1,74	1,53	
5	15,2	11,56	8,34	5,96	4,38	3,3	2,73	2,4	2,3	
10	15,2	11,7	8,7	6,55	5,45	4,82	4,65	4,4	4,54	
w/out comp	12,7	9,34	6,75	4,8	3,4	2,38	1,71	---	--	E_{2v}
1	12,7	9.34	6,76	4,81	3.41	2,39	1,72	---	...	
3	12,7	9,35	6,77	4,83	3,48	2,54	1,98	--	—	
5	12,73	9,36	6,85	5,0	3,73	2,91	2,6	—	—	
10	12,78	9,7	7,38	5,92	5,15	4,76	4,65	--	—	
1	0,11	0,13	0,13	0,15	0,18	0,21	0,25	0,3	0,34	E_{edge}
3	0,55	0,58	0,69	0,78	0,92	1,07	1,2	1,37	1,38	
5	1,18	1,33	1,44	1,58	1,9	2,11	2,25	2,37	2,3	
10	2,2	3,4	2,83	4,2	4,47	4,52	4,64	4,5	4,53	

TABLE II

Interpolator	$R(\tau) = exp(-\alpha/\|\tau\|)$									
τ / Z%	1	2	4	8	16	32	64	128	256	E%
w/out comp.	15,2	11,5	8 33	5,8	4,16	2 91	2,05	1.43	1,05	$E_{0,5}$
1	15,2	11,5	8,41	5,82	4,18	2,95	2,11	1,51	1,15	
3	15,2	11,51	8,49	5,91	4,35	3,25	2,65	2,30	2,15	
5	15,2	11,52	8,68	5,17	4,93	4,02	3,41	3,55	3,5	
10	15,6	12,1	9,87	8.08	7,4	7,1	7,01	6,88	6,91	
w/out comp.	12,7	9,34	6,75	4,76	3,4	2,38	1,71	--	---	E_{av}
1	12,63	9,34	6,87	4,8	3,42	2,43	1,14	-	—	
3	12,67	9,32	6,94	4,93	3,71	2,35	2,43	-	---	
5	12,72	9,42	7,14	5,4	4,46	3,86	3,64			
10	13,0	10,57	8,8	7,75	7,27	7,05	7,0	--	--	
1	0,213	0,23	0,253	0,313	0,376	0,44	0,5	0,57	0,63	E_{edge}
3	1,15	1,18	1,39	1,57	1,59	1,75	2,08	2,09	2,1	
5	2,13	2,43	2,62	3,03	3,3	3,43	3,53	3,5	3,49	
10	5,93	6,26	6,50	6,98	7,06	7,02	6,9	6,88	6,92	

TABLE III

Predictor	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
w/out comp.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
w/out comp.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	E_{av}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	E_{edge}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

TABLE IV

Interpolator	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
w/out comp.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
w/out comp.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			E_{av}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	E_{edge}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

TABLE V /16

Predictor

K_{comp}

TABLE VI /17

Interpolator

K_{comp}

TABLE VII

Predictor

	[illegible]	2	[illegible]	[illegible]	16	[illegible]	[illegible]
1	1.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	1.15	[illegible]	1.66	2.20	[illegible]	[illegible]
5	1.2	1.3	[illegible]	[illegible]	5.6	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	8.5	[illegible]	[illegible]

K_{comp}

TABLE VIII

Interpolator

%	1	2	4	8	16	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	4.8	[illegible]	[illegible]
3	1.45	[illegible]	2.03	4.4	9.5	18.2	[illegible]
5	1.22	1.54	2.07	6.15	12.7	[illegible]	[illegible]
10	1.77	2.32	3.04	9.2	15.5	[illegible]	[illegible]

K_{comp}

TABLE IX

3%

Method of comp.	ZOP			IFO	
γ	Thresh. const	Thresh	2%	Thresh const.	Thresh
[illegible]	22,[illegible]	[illegible]	23,6	[illegible]	[illegible]
[illegible]	6,0	6,0	22.0	20.[illegible]	20 0
[illegible]	3,8	5.3	3,4	[illegible]	3,9
8	2.3	3,5	3,2	[illegible]	5,2
[illegible]	2,6	2,2	2,6	2	2,5

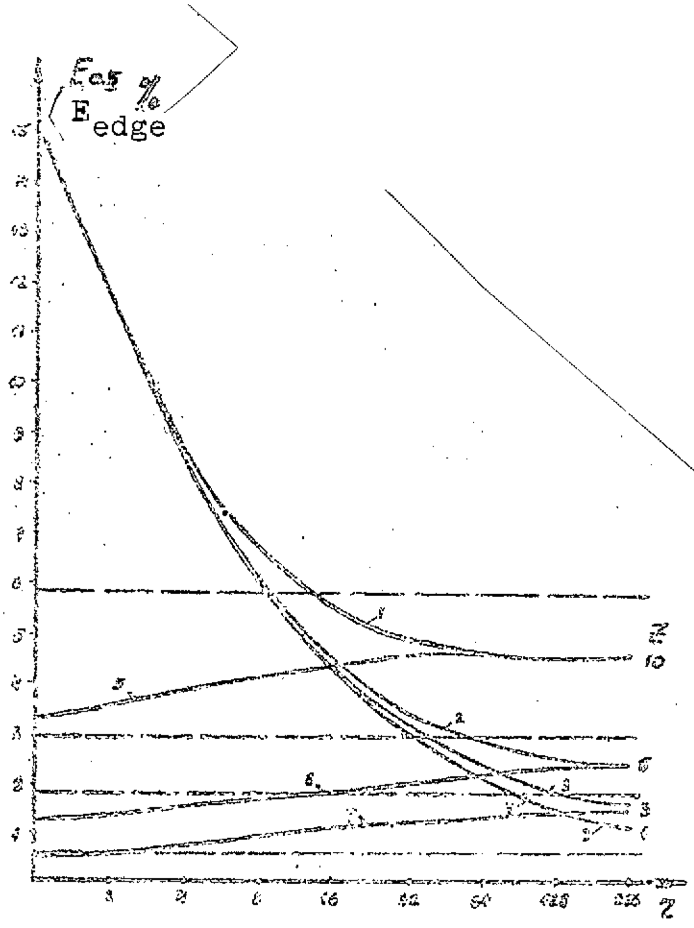

Fig. 1. Error with predictor. First model of signal. 1,2,3,4,8--$E_{0.5}$, 5,6,7--E_{edge}, 8--without compression.

/19

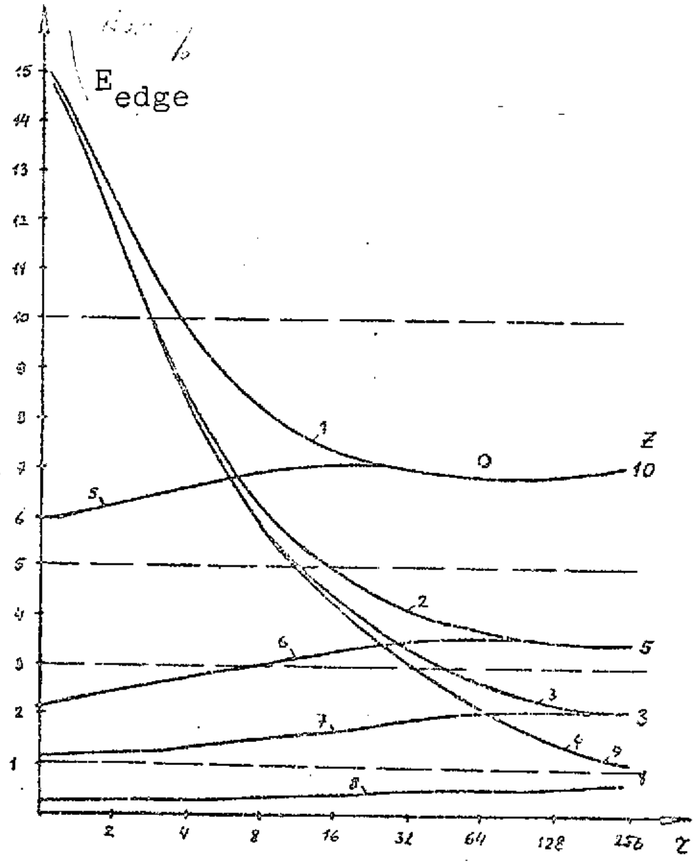

Fig. 2. Errors with interpolator, first model of signal. 1,2,3,4,9--$E_{0.5}$, 5,6,7,8--E_{edge}, 9--without compression.

/20

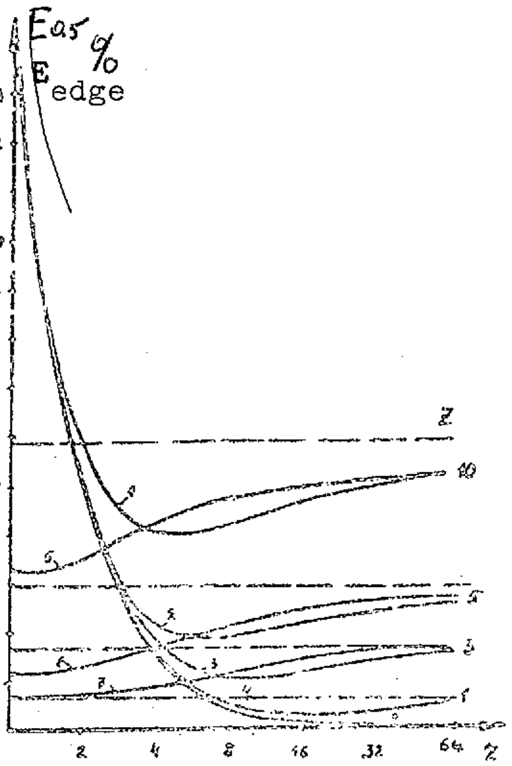

Fig. 3. Errors with predictor, second model of signal. 1,2,3,4,8--$E_{0.5}$, 5,6,7--E_{edge}, 8--without compression.

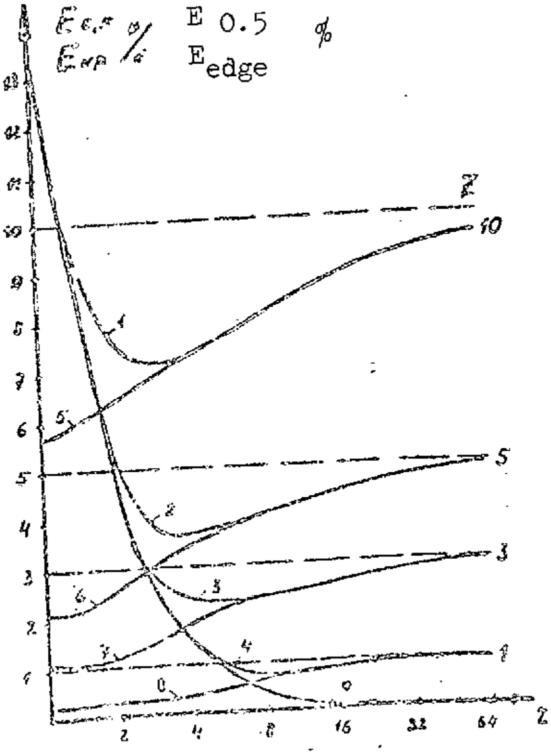

Fig. 4. Errors with interpolator, second model of signal. 1,2,3,4,9--$E_{0.5}$, 5,6,7,8--E_{edge}, 9--without compression.

/21

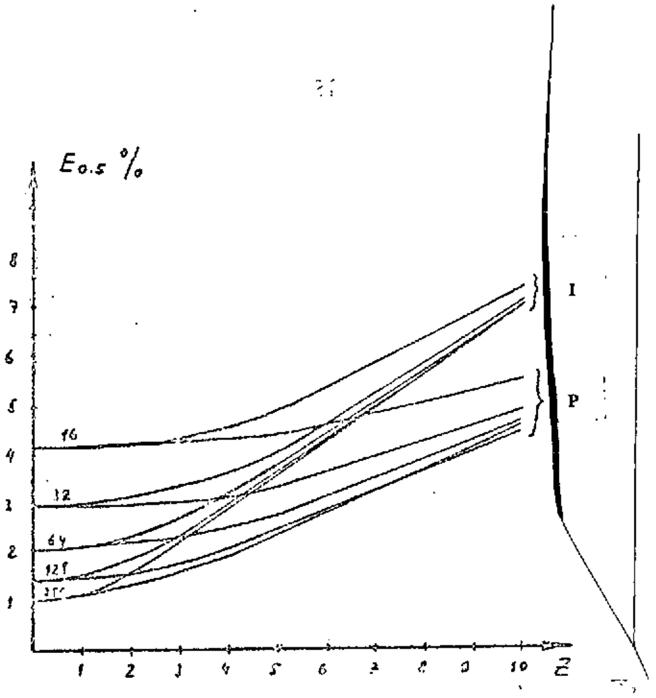

Fig. 5. Error as a function of threshold first signal model.

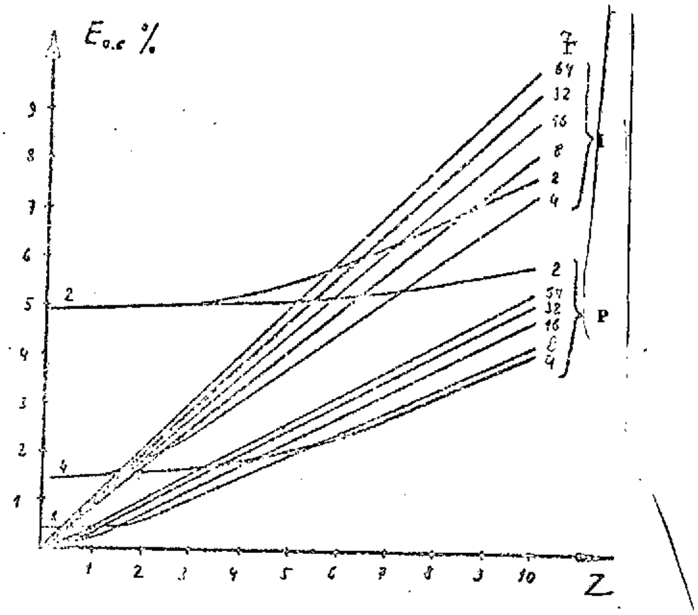

Fig. 6. Error v. threshold, second signal model.

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